Math 344: Quiz 3 Solutions

Fall 2018

For each differential equation below, determine the following:

- the **standard form** of the equation;
- whether $x_0 = 0$ is an **ordinary**, **regular singular**, or **irregular singular** point; and
- whether the general solution can be written in the form $y(x) = c_1 y_1(x) + c_2 y_2(x)$, where:
  - **both** $y_1(x), y_2(x)$ can be represented by **power series**; or
  - **both** $y_1(x), y_2(x)$ can be represented by **Frobenius series**; or
  - **at least one** of $y_1(x), y_2(x)$ can be represented by a **Frobenius series**; or
  - the form of $y_1(x), y_2(x)$ is **unknown**.

Differential Equation	Standard Form	Point Type	Form of $y_1(x), y_2(x)$
$(x^2-3)y'' - 3xy' - 5y = 0$	$y'' - \frac{3x}{x^2-3}y' - \frac{5}{x^2-3}y = 0$	ordinary	power series
$x^2y'' + x\cos(x)y' - 2e^x y = 0$	$y'' + \frac{\cos(x)}{x}y' - \frac{2e^x}{x^2}y = 0$	regular singular	both Frobenius $r = \pm\sqrt{2}$
$(1-4x^2)y'' - 20xy' - 16y = 0$	$y'' - \frac{20x}{1-4x^2}y' - \frac{16}{1-4x^2}y = 0$	ordinary	power series
$x^2y'' + 2x^2y' + (x - \frac{3}{4})y = 0$	$y'' + 2y' + \frac{x-\frac{3}{4}}{x^2}y = 0$	regular singular	≥ 1 Frobenius $r = \frac{3}{2}, -\frac{1}{2}$
$3xy'' + \frac{1}{x}y' - \frac{2x-1}{x}y = 0$	$y'' + \frac{1}{3x^2}y' - \frac{2x-1}{3x^2}y = 0$	irregular singular	unknown
$xy'' - xy' + y = 0$	$y'' - y' + \frac{1}{x}y = 0$	regular singular	≥ 1 Frobenius $r = 1, 0$
$2xy'' + y' - 2xy = 0$	$y'' + \frac{1}{2x}y' - y = 0$	regular singular	both Frobenius $r = \frac{1}{2}, 0$