Math 344: Quiz 4 Solutions

Fall 2018

Problem 1. For each function below, determine its fundamental period (if it has one).

a) $f(x) = 1 - \cos(6\pi x) + \sin(4\pi x)$

b) $g(x) = 3\cos(2x) + 2\sin(\pi x)$

c) $h(x) = (1-3i)e^{-4\pi ix} + 2 - e^{\pi ix}$

Solution.

a) The function $f_1(x) = 1$ has periods every $T > 0$. The function $f_2(x) = -\cos(6\pi x)$ has periods $T = \frac{1}{3}, \frac{2}{3}, 1, \ldots$. The function $f_3(x) = \sin(4\pi x)$ has periods $T = \frac{1}{2}, 1, \frac{3}{2}, \ldots$. The least common period of these three functions is therefore $T = 1$, and so that is the fundamental period of f.

b) The function $g_1(x) = 3\cos(2x)$ has periods $T = \pi, 2\pi, 3\pi, \ldots$. The function $g_2(x) = 2\sin(\pi x)$ has periods $T = 2, 4, 6, \ldots$. Since these two functions do not share a common period, the function g is not periodic.

c) The function $h_1(x) = (1-3i)e^{-4\pi ix}$ has periods $T = \frac{1}{2}, 1, \frac{3}{2}, \ldots$. The function $h_2(x) = 2$ has periods every $T > 0$. The function $h_3(x) = -e^{\pi ix}$ has periods $T = 2, 4, 6, \ldots$. The least common period of these three functions is $T = 2$, so that is the fundamental period of h.

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Problem 2. Let $f(x) = 2\sin(2\pi x) - \cos(8\pi x) + 5\sin(8\pi x)$. Determine the complex Fourier series representation of f.

Solution. Using the identities $\cos(\theta) = \frac{e^{i\theta}+e^{-i\theta}}{2}$ and $\sin(\theta) = \frac{e^{i\theta}-e^{-i\theta}}{2i}$, we can write

$$\begin{aligned} f(x) &= 2\left(\frac{e^{2\pi ix} - e^{-2\pi ix}}{2i}\right) - \left(\frac{e^{8\pi ix} + e^{-8\pi ix}}{2}\right) + 5\left(\frac{e^{8\pi ix} - e^{-8\pi ix}}{2i}\right) \\ &= \left(-\frac{1}{2} - \frac{5}{2i}\right)e^{-8\pi ix} - \frac{1}{i}e^{-2\pi ix} + \frac{1}{i}e^{2\pi ix} + \left(-\frac{1}{2} + \frac{5}{2i}\right)e^{8\pi ix} \\ &= \left(-\frac{1}{2} + \frac{5}{2}i\right)e^{-8\pi ix} + ie^{-2\pi ix} - ie^{2\pi ix} + \left(-\frac{1}{2} - \frac{5}{2}i\right)e^{8\pi ix} \end{aligned}$$

(We used the identity $\frac{1}{i} = -i$ to simplify the coefficients in the last line, but that wasn't necessary.) ■