Vocabulary

- We say the **limit of** $f(x)$ **as** x **approaches** a **is** L if

  In terms of notation, we write

- A function f is **continuous at** a if

  1)

  2)

  3)

- The **derivative** of a function f at a is

  In terms of the graph of f, this number is the __________ of the __________ line to the graph at the point $(a, f(a))$.

Concepts

1. Sketch the graph of a function f that has the following properties:

$$\lim_{x\to 0} f(x) = \infty, \quad f(1) = -1, \quad \lim_{x\to 1^-} f(x) = 0, \quad \lim_{x\to 1^+} f(x) = -2, \quad \lim_{x\to\infty} f(x) = 1$$

2. Sketch the graph of a continuous function that has a horizontal asymptote at $y = 1$ that the graph crosses exactly three times.

3. If $\lim_{x\to\infty} f(x) = 4$, then the graph has a _____________ asymptote at _____________.

4. Sketch the graph of a function that is defined everywhere but is not continuous at $x = 1$.

5. Sketch the graph of a function f that is continuous on $[1,3]$ but not differentiable at $x = 2$.

6. **True** or **False**: If $\lim_{x\to 2} f(x) = 3$, then necessarily $f(2) = 3$.

7. **True** or **False**: If $f(2) = 3$, then necessarily $\lim_{x\to 2} f(x) = 3$.

8. **True** or **False**: If f is continuous, then necessarily $\lim_{x\to 2} f(x) = f(2)$.

9. **True** or **False**: If f is continuous at a, then f must be differentiable at a.

10. **True** or **False**: If f is differentiable at a, then f must be continuous at a.

Computations

Limits

Exercise 1. Compute the following limits. If a limit does not exist, briefly explain why.

a) $\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x}$ **b)** $\lim_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x}$ **c)** $\lim_{x\to -\infty} \frac{x^2-7x}{x+1}$ **d)** $\lim_{x\to 3^+} \frac{x^2+4}{x-3}$

ADDITIONAL PRACTICE

- §2.2: 11-42
- §2.6: 13-36, 69-76

Continuity

Exercise 2. If f is continuous and $3x-3 \leq f(x) \leq x^2-x+1$ for every $x \neq 2$, find $\lim_{x\to 2} f(x)$.

Exercise 3. Where is the function $f(x) = \frac{\sin(x)}{x^2-1}$ continuous?

Exercise 4. Determine the value of a that makes the function below continuous at 1:

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{for } x \neq 1 \\ a, & \text{for } x = 1. \end{cases}$$

ADDITIONAL PRACTICE

- §2.5: 29, 30, 37-46

Derivatives

Exercise 5. For each function below, use the limit definition of the derivative to find an equation for the line tangent to the graph at the given point:

a) $f(x) = 1 - 2x^2 \quad (1, -1)$ **b)** $g(x) = -x^3, \quad (-2, 8)$ **c)** $F(t) = \frac{3}{t^2}, \quad (3, \frac{1}{3})$

ADDITIONAL PRACTICE

- §3.1: 5-22

Exercise 6. Compute each function's derivative using the limit definition of the derivative:

a) $f(x) = x^2 + 2$ **b)** $g(x) = \sqrt{x+1}$ **c)** $F(x) = \frac{1}{x-1}$

ADDITIONAL PRACTICE

- §3.2: 7-26

Differentiation Rules

The Power Rule $\frac{\mathrm{d}}{\mathrm{d}x}(x^n) =$

Trig Rules $\frac{\mathrm{d}}{\mathrm{d}x}(\sin(x)) =$

$\frac{\mathrm{d}}{\mathrm{d}x}(\cos(x)) =$

The Product Rule $(f(x)g(x))' =$

The Quotient Rule $\left(\frac{f(x)}{g(x)}\right)' =$

The Chain Rule $(f(u(x)))' =$

Exercise 7. Compute the derivative of each function below.

a) $f(x) = x^2\cos(3x-1)$ **b)** $g(t) = \left(\sin(t^2-1)\right)^8$ **c)** $F(\theta) = \frac{\sin(\theta^3)}{\theta^2+1}$

ADDITIONAL PRACTICE

- Chapter 3 Review: 1-7, 9-13, 17-20, 27-40

Implicit Differentiation

Exercise 8. Find the equation of the line tangent to the curve $2x^4 + y^3 = 5x^2 y$ at the point $(1,2)$.

Exercise 9. If $\sqrt{x} + \sqrt{y} = 1$, find $\frac{d^2 y}{dx^2}$.

ADDITIONAL PRACTICE

- §3.7: 1-28
- Chapter 3 Review: 41-48